



Oxford Cambridge and RSA

Thursday 6 June 2019 – Afternoon

A Level Further Mathematics B (MEI)

Y432/01 Statistics Minor

Time allowed: 1 hour 15 minutes



You must have:

- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You may use:

- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [].
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

Answer **all** the questions.

- 1** In a game at a charity fair, a spinner is spun 4 times.
On each spin the chance that the spinner lands on a score of 5 is 0.2.
The random variable X represents the number of spins on which the spinner lands on a score of 5.
- (a) Find $P(X = 3)$. [2]
- (b) Find each of the following.
- $E(X)$
 - $\text{Var}(X)$ [2]
- One game costs £1 to play and, for each spin that lands on a score of 5, the player receives 50 pence.
- (c) (i) Find the expected total amount of money gained by a player in one game. [2]
- (ii) Find the standard deviation of the total amount of money gained by a player in one game. [1]
- 2** A market researcher wants to interview people who watched a particular television programme. Audience research data used by the broadcaster indicates that 12% of the adult population watched this programme. This figure is used to model the situation.
The researcher asks people in a shopping centre, one at a time, if they watched the programme. You should assume that these people form a random sample of the adult population.
- (a) Find the probability that the fifth person the researcher asks is the first to have watched the programme. [2]
- (b) Find the probability that the researcher has to ask at least 10 people in order to find one who watched the programme. [1]
- (c) Find the probability that the twentieth person the researcher asks is the third to have watched the programme. [3]
- (d) Find how many people the researcher would have to ask to ensure that there is a probability of at least 0.95 that at least one of them watched the programme. [3]
- 3** A company has been commissioned to make 50 very expensive titanium components.
A sample of the components needs to be tested to ensure that they are sufficiently strong. However, this is a test to destruction, so the components which are tested can no longer be used.
- (a) Explain why it would not be appropriate to use a census in these circumstances. [1]
- A manager suggests that the first 5 components to be manufactured should be tested.
- (b) Explain why this would not be a sensible method of selecting the sample. [1]
- A statistician advises the manager that the sample selected should be a random sample.
- (c) Give two desirable features (other than randomness) that the sample should have. [2]

- 4 Zara uses a metal detector to search for coins on a beach. She wonders if the numbers of coins that she finds in an area of 10m^2 can be modelled by a Poisson distribution. The table below shows the numbers of coins that she finds in randomly chosen areas of 10m^2 over a period of months.

| | | | | | | | | |
|-----------------------|----|----|----|----|----|---|---|------|
| Number of coins found | 0 | 1 | 2 | 3 | 4 | 5 | 6 | >6 |
| Frequency | 13 | 28 | 30 | 14 | 10 | 2 | 3 | 0 |

- (a) Software gives the sample mean as 1.98 and the sample standard deviation as 1.4212. Explain how these values suggest that a Poisson distribution may be an appropriate model for the numbers of coins found. [2]

Zara decides to carry out a chi-squared test to investigate whether a Poisson distribution is an appropriate model.

Fig. 4 is a screenshot showing part of the spreadsheet used to analyse the data. Some values in the spreadsheet have been deliberately omitted.

| | A | B | C | D |
|---|-----------------------|--------------------|--------------------|--------------------------|
| 1 | Number of coins found | Observed frequency | Expected frequency | Chi-squared contribution |
| 2 | 0 | 13 | 13.8069 | 0.0472 |
| 3 | 1 | 28 | | |
| 4 | 2 | 30 | 27.0643 | 0.3184 |
| 5 | 3 | 14 | 17.8625 | 0.8352 |
| 6 | 4 | 10 | 8.8419 | 0.1517 |
| 7 | ≥ 5 | 5 | | 0.0015 |
| 8 | | | | |

Fig. 4

- (b) Showing your calculations, find the missing values in each of the following cells.
- C3
 - C7
 - D3
- [4]
- (c) Explain why the numbers for 5, 6 and more than 6 coins found have been combined into the single category of at least 5 coins found, as shown in the spreadsheet. [1]
- (d) Complete the hypothesis test at the 5% level of significance. [6]

For the rest of this question, you should assume that the number of coins that Zara finds in an area of 10m^2 can be modelled by a Poisson distribution with mean 1.98.

Zara also finds pieces of jewellery independently of the coins she finds. The number of pieces of jewellery that she finds per 10m^2 area is modelled by a Poisson distribution with mean 0.42.

- (e) Find the probability that Zara finds a total of exactly 3 items (coins and/or jewellery) in an area of 10m^2 . [2]
- (f) Find the probability that Zara finds a total of at least 30 items (coins and/or jewellery) in an area of 100m^2 . [2]

- 5 A student wants to know if there is a positive correlation between the amounts of two pollutants, sulphur dioxide and PM10 particulates, on different days in the area of London in which he lives; these amounts, measured in suitable units, are denoted by s and p respectively. He uses a government website to obtain data for a random sample of 15 days on which the amounts of these pollutants were measured simultaneously. Fig. 5.1 is a scatter diagram showing the data. Summary statistics for these 15 values of s and p are as follows.

$$\Sigma s = 155.4 \quad \Sigma p = 518.9 \quad \Sigma s^2 = 2322.7 \quad \Sigma p^2 = 21\,270.5 \quad \Sigma sp = 6009.1$$

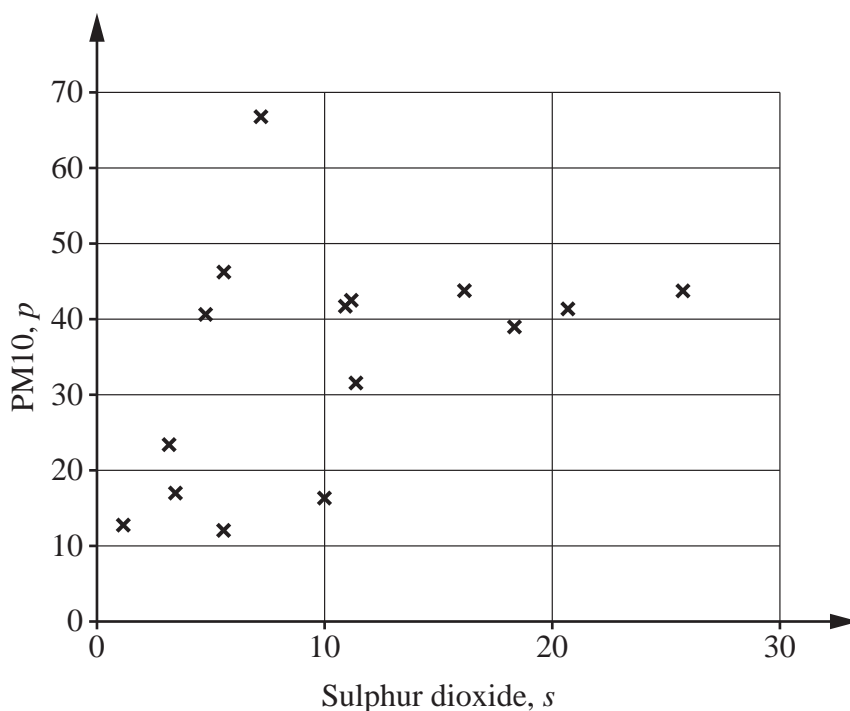


Fig. 5.1

- (a) Explain why the student might come to the conclusion that a test based on Pearson's product moment correlation coefficient may be valid. [2]
- (b) Find the value of Pearson's product moment correlation coefficient. [4]
- (c) Carry out a test at the 5% significance level to investigate whether there is positive correlation between the amounts of sulphur dioxide and PM10 particulates. [5]
- (d) Explain why the student made sure that the sample chosen was a random sample. [2]

The student also wishes to model the relationship between the amounts of nitrogen dioxide n and PM10 particulates p .

He takes a random sample of 54 values of the two variables, both measured at the same times. Fig. 5.2 is a scatter diagram which shows the data, together with the regression line of n on p , the equation of the regression line and the value of r^2 .

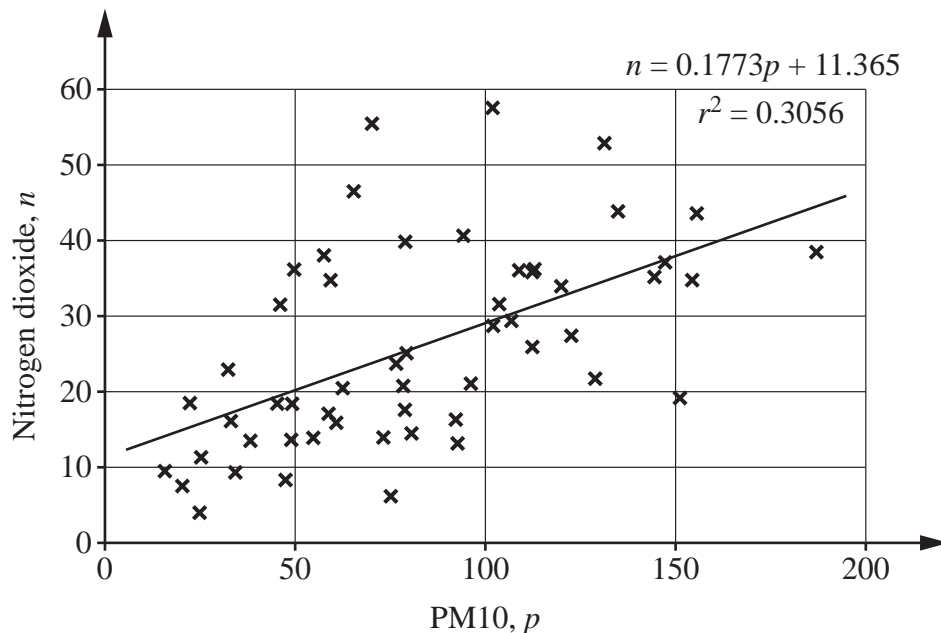


Fig. 5.2

(e) Predict the value of n for $p = 150$. [1]

(f) Discuss the reliability of your prediction in part (e). [2]

6 The discrete random variable X has a uniform distribution over $\{n, n + 1, \dots, 2n\}$.

(a) Given that n is odd, find $P(X < \frac{3}{2}n)$. [1]

(b) Given instead that n is even, find $P(X < \frac{3}{2}n)$, giving your answer as a single algebraic fraction. [3]

(c) The sum of 6 independent values of X is denoted by Y . Find $\text{Var}(Y)$. [3]

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